Math Counts:
Issues That Matter

A Professional Series, Volume 6

MASTERING THE BASIC FACTS

Understanding and learning basic facts to automaticity is a primary cornerstone in mathematics learning. The ability to recall basic facts automatically from long term memory makes solving more complex problems, such as basic operation multistep problems, more efficient and less likely to result in errors (Geary and Widaman). Many questions revolve around the mastery of basic facts. What are the basic facts? How can basic facts be taught to help students understand the operations while also helping them memorize the basic facts? What is more important, memorization of basic facts or conceptual understanding?

What are the basic facts?

The basic facts for addition and multiplication are all single digit addends and factors. For example, 8 x 2 = 16 and 7 + 4 = 11 are basic facts but 16 x 2 = 32 is not a basic fact. Subtraction basic facts are the inverses of all addition basic facts. So, since 7 + 4 = 11 is a basic addition fact, 11 – 4 = 7 and 11 – 7 = 4 are basic subtraction facts. Likewise, basic division facts are the inverses of basic multiplication facts. For example, since 8 x 7 = 56 is a basic multiplication fact, 56 ÷ 7 = 8 and 56 ÷ 8 = 7 are basic division facts.

More rigorous mathematics programs may include addition facts to sums of 20 rather than 18, their subtraction inverses, multiplication facts with factors to 12 rather than 9, and their division inverses. Such rigor is a benefit in any math program since it allows students to have a broader range of facts at their disposal. Further, a larger range of facts to learn means that students increase the depth and complexity of their fact knowledge while developing their understanding of the meaning of each operation.

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How do students learn basic facts?

Students need to learn basic facts in the early grades using a format that fosters both mastery and understanding. "Basic computational and procedural skills develop over time" and require sustained, repeated practice to be mastered (California Framework 9). Ideally, basic fact chapters are clustered so that an extended amount of time is spent on facts, without interruption by other topics such as geometry or data. However, when introducing the addition and subtraction facts through sums and differences of 20, it is wise to pause after sums and differences to 12 and give children time to internalize them before introducing the final batch of facts.

All along, daily fact practice should use an approach that aids students in distinguishing among the different operations and understanding the different meanings of each operation. Students who understand the various meanings of an operation, use thinking strategies for fact recall, and then proceed to drill and practice will be well equipped for success in math as they move through the grades. Students who memorize facts without understanding the meanings of the operations often are not sure when or how to use the facts. Such learning without understanding is often fragile and can easily breakdown (NCTM). Rote fact memorization through only drill and practice is the least valuable way to learn facts.

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Meanings of each operation:

- **Addition**

  Joining is combining two smaller sets into a larger set.

- **Subtraction**

  Comparison is finding out which group has more or less by matching objects.

  Taking away is removing items from a larger group to form a smaller group.

  Separating is breaking one larger group into two smaller groups.

- **Multiplication**

  Equal groups uses multiplication to find the total number in groups of the same size.

  Repeated addition multiplication is a shortcut for adding same-sized groups.

  Combinations use multiplication to find the total number of ways two or more items can be combined.

- **Division**

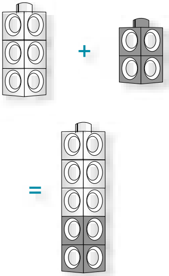
  Fair shares breaks a larger group into equal-sized groups, or equal shares. The fair shares can show the size of each share.

  Repeated subtraction is a shortcut for subtracting same-sized groups.

Students learn the basic facts in three stages: the manipulative stage, the pictorial stage, and the symbolic stage. Each stage builds upon the previous stage to help students master their basic facts.

Concrete Stage

First, students are introduced to the various meanings of each operation through manipulatives. Students concretely work with manipulatives to model the action of each operation. By modeling the action of a problem students physically act out each operation, thereby gaining an in depth understanding of the meanings of each operation.



For example, an addition fact.

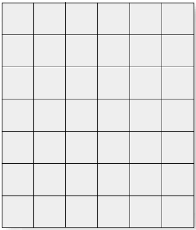
3 + 2 = _

Students model **joining** as a meaning of addition by joining the two groups of connecting cubes.

3 + 2 = 5

Pictorial Stage

Next, students move from manipulative models to pictorial representations of each basic operation. In the pictorial stage students become more proficient in performing the operation with a pictorial aide to support them in the transition from concrete manipulatives to the symbolic representations of basic facts.



For example, a multiplication fact.

6 x 7 = _

Students see the two factors represented in each row and column of the array and can find the product using the array.

6 x 7 = 56

Symbolic Stage

Finally, students move to a purely symbolic representation. At this point, students do not use models or pictorial representations of the procedures. They see the numerals and the operation sign and must determine the answer to the fact.

For example, a division fact.

72 ÷ 8 = _

Students see the equation and solve it. At first students may use strategies or properties to solve the equation. Later, the fact should be mastered and recalled from memory.

72 ÷ 8 = 9

Strategies and Properties for the Basic Operations

Addition

Counting on: students start with the larger addend and then count on the smaller addend, generally 1, 2, or 3.

Doubles plus one: students take a known doubles fact and add one when one addend is one more than the other addend.

Commutative Property: students may change the order of the addends without changing the sum.

Associative Property: students can group numbers in any order when adding more than two numbers.

Subtraction

Counting Back: students start with the greater number and then count back the smaller number, generally 1, 2, or 3.

Doubles: students relate a subtraction problem to a known doubles fact in addition to find the difference.

Related Addition Facts: students use known addition facts to help them find subtraction facts.

Multiplication

Repeated Addition: students can repeatedly add the second factor the number of times of the first factor

Skip Counting: students use number lines to skip count the second factor by the number of times shown in the first factor.

Associative Property: when multiplying more than two factors students may group together any pair of factors without changing the final product.

Commutative Property: students may change the order of the factors without changing the product.

Zero Property: students can multiply any number by zero with the result always being zero.

Related Multiplication Facts: students use known multiplication facts to help them find division facts.

Identity Property: students can multiply any number by one with the result always being the number.

Division

Repeated Subtraction: students can repeatedly subtract the divisor from the dividend until the dividend reaches 0 or a number less than the divisor.

Skip Counting: students use number lines to skip count backwards on number lines starting with the dividend and skipping back groups of the divisor.

Zero Property: students can divide 0 by any number and the quotient is always 0.

After students have had ample time using various strategies and properties, practice sessions are essential to build up speed and accuracy. In the practice sets, the problems must be varied. Students should not always practice just horizontal problems or problems where the greater number is always the first number. A variety of problem formats leads to students having a fuller understanding of addition, subtraction, multiplication, and division facts. A strong mathematics program will provide fact drill pages in the student book, technology that supports fact mastery, and independent tools students can use to practice facts. In addition, feedback is essential for students to improve their fact mastery. Without "feedback, learning and improvement is minimal, even for highly motivated students" (Ericsson 365). Whether the feedback comes from teachers who suggest alternative fact strategies or technology which identifies a student's errors it must be consistent and informative to help students improve their facts.

"Both basic facts and conceptual understanding need to be developed together."

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One point to remember as children learn their basic facts is that not all facts are at the same level of difficulty. When students first use strategies and properties to learn the easiest facts, they can then apply the strategies and properties to the more difficult facts. The following charts organize the facts by their level of difficulty.

As students are learning the basic facts, an inventory can help keep track of the facts students have not yet learned so they can focus their efforts on learning the facts they do not know. Extra practice options on the basic facts are valuable for students so they have several ways to practice the basic facts, especially the ones they have not memorized to automaticity.

What is more valuable, basic facts or conceptual understanding?

As mathematics educators we often debate between teaching students to memorize basic facts or to develop conceptual understanding. Some have thought that automaticity with basic fact runs counter to attainment of conceptual understanding. In fact, both basic facts and conceptual understanding need to be developed together. Conceptual understanding of more complex mathematical concepts is built on understanding of basic facts. At the same time a student must have an understanding of the meanings and uses of basic skills to proceed onto more advanced topics. Analogous to this event is the dancer who worries about foot positions cannot hope to impress and audience with the grace and elegance of his or her technique. Good mathematics depends on a blend of basics facts and conceptual understanding.

Conclusions

There are three key reasons to teach basic facts to automaticity (adapted from Wu):

1. Basic facts are **absolutely indispensable** for the understanding of more sophisticated processes. Students will not be as successful with mathematical problems such as multidigit exercises, finding area, and solving word problems if they have not mastered the basic facts.

2. The automaticity of basic facts frees up mental energy to concentrate on the more rigorous demands of a complicated problem. When a student knows his or her basic facts to automaticity, the student can focus his or her efforts on how to properly address and solve the problem rather than getting caught up in remembering the basic facts.

3. When a skill is bypassed in favor of a solely conceptual approach, the resulting conceptual understanding is often superficial. In other words, seeing the big picture without noticing the fine details does not necessarily lead to a solid, complete understanding of the basic facts.

Basic fact mastery is a foundation for deep math understanding. Students who understanding the meanings of each operation and master the basic facts have a strong, solid foundation on which they can develop more complex mathematical skills, attain a broader conceptual understanding of mathematics, and become superior problem solvers.

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