Geometry and Measurement of Plane Figures

Activity Set 5

Trainer Guide

NGSSS 7.A.5.2
NGSSS 8.G.2.4

Pythagoras Who?

In this activity, participants will use the Pythagorean theorem to find the side lengths of a right triangle and to solve problems.

MATERIALS

- *Transparency/Page: Right Triangles (Directions)*
- *Transparency/Page: 1-cm Grid*
- *Transparency/Page: $\frac{1}{2}$-cm Grid*
- *Transparency/Page: Right Triangle Relations*
- *Transparency/Page: Right Triangle Relations Answer Key*
- *Transparency/Page: Right Triangle Reasoning*
- *Transparency/Page: Right Triangle Reasoning Answer Key*
- blank transparency
- ruler (1 for each participant)
- protractor (1 for each participant)
- scissors (1 for each participant)
- 1 measuring tape
- pencils
- calculators (1 for each group)

VOCABULARY

- hypotenuse
- leg
- Pythagorean theorem
- right triangle

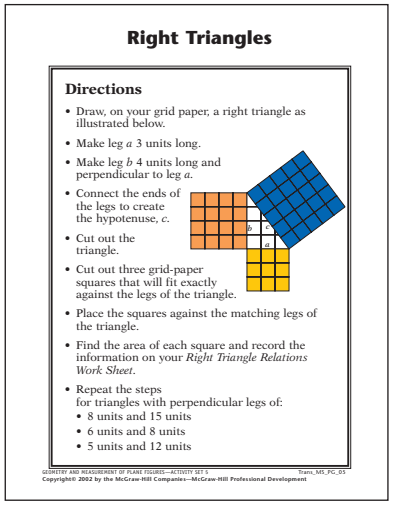
TIME: 40 minutes

INTRODUCE

- Explain to participants that they are going to learn about the Pythagorean theorem, a statement of the relationship that exists among the sides of a right triangle.
- Ask participant volunteers to use the measuring tape to find the length and width of the classroom, white board, or other specified rectangular object that has a right-angle corner.
- Draw a similar rectangle on a blank transparency and record the measurements as the volunteers report them to the class.
- Ask participants if it is possible to find the diagonal length of a rectangle without measuring it. (It is possible, using the Pythagorean theorem.)
- Explain that at the end of the activity the group will apply what it has learned to do exactly that.

DISCUSS AND DO

- Explain to participants that they are going to work with a model that illustrates the Pythagorean theorem.
- Display *Transparency: Right Triangles Directions*.
- Go over the steps for the activity.

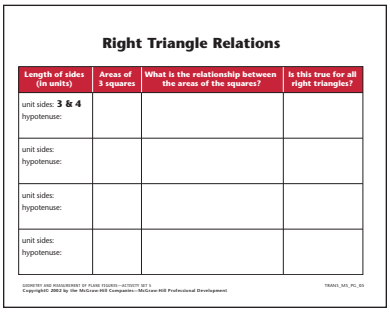

Right Triangles

Directions

- Draw, on your grid paper, a right triangle as illustrated below.
- Make leg *a* 3 units long.
- Make leg *b* 4 units long and perpendicular to leg *a*.
- Connect the ends of the legs to create the hypotenuse, *c*.
- Cut out the triangle.
- Cut out three grid-paper squares that will fit exactly against the legs of the triangle.
- Place the squares against the matching legs of the triangle.
- Find the area of each square and record the information on your *Right Triangle Relations Work Sheet*.
- Repeat the steps for triangles with perpendicular legs of:
  - 8 units and 15 units
  - 6 units and 8 units
  - 5 units and 12 units

Transparency: Constructing Right Triangles Directions

TEACHING TIP: Instruct participants to use their *1-cm Grid* pages for the first triangle and its squares, and $\frac{1}{2}$*-cm Grid* pages for the remaining three triangles and the corresponding squares.



Right Triangle Relations

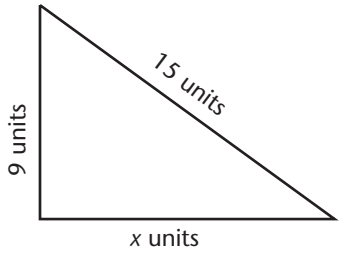
Length of sides (in units)	Areas of 3 squares	What is the relationship between the areas of the squares?	Is this true for all right triangles?
unit sides: **3 & 4** hypotenuse:			
unit sides: hypotenuse:			
unit sides: hypotenuse:			
unit sides: hypotenuse:			

Transparency: Right Triangle Relations

- Ask participants to work in groups of three.
- Allow about 12–15 minutes for them to complete the activity.
- Have volunteers report the findings of their groups. Record the results on *Transparency: Right Triangle Relations*. (Refer to *Transparency: Right Triangle Relations Answer Key* to resolve any questions.)
- Have participants take out their first grid paper models.
- Have them cut the two smaller squares in such a way that the pieces fit into the larger square.
- Ask them, after a few minutes, to describe the relationship between the two areas (e.g., Do the smaller squares together cover the larger square; is there space left over; are there pieces of the smaller squares left over?). (The pieces of the two smaller squares exactly cover the larger square.)
- Ask participants to use what they have just done to describe the relationship between the areas of the squares of the two legs of a triangle and the area of the square of the hypotenuse. (The sum of the squares of the legs is equal to the square of the hypotenuse: $a^2 + b^2 = c^2$.)
- Express the relationship as $a^2 + b^2 = c^2$, if participants do not mention it.
- Ask participants if this relationship holds true for all triangles.
- Explain that it works only for right triangles, if they do not mention this.

CONCLUDE

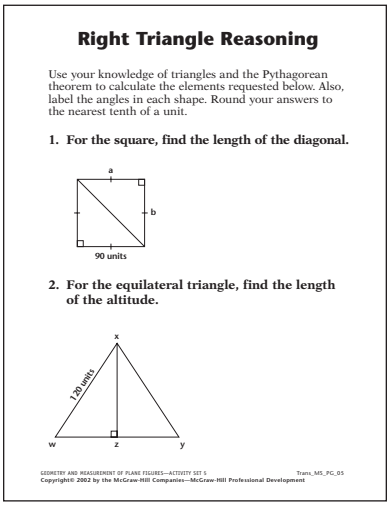
- Draw, on a blank transparency, a right triangle in the proportions and with the labels shown below.



- Ask participants how they could use what they have learned about the Pythagorean theorem to find the measure of x.
- Explain, if they do not mention it, that they can use the formula $a^2 + b^2 = c^2$, fill in the known elements, and solve for the unknown.
- Go through the steps.

$$a^2 + b^2 = c^2$$
$$9^2 + b^2 = 152$$
$$81 + b^2 = 225$$
$$b^2 = 144$$
$$b = 12$$

- Ask participants how they would find b if they know what b^2 is.
- Remind them that taking the square root of a squared number will return the original number.
- Point out that this example included numbers that came out evenly, but that this is not always the case.
- Display *Transparency: Right Triangle Reasoning* and have participants take out their matching pages.
- Have participants work in groups of three to answer the questions on their pages.



Right Triangle Reasoning

Use your knowledge of triangles and the Pythagorean theorem to calculate the elements requested below. Also, label the angles in each shape. Round your answers to the nearest tenth of a unit.

1. For the square, find the length of the diagonal.

a
b
90 units

2. For the equilateral triangle, find the length of the altitude.

x
120 units
w
z
y

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Transparency: Right Triangle Reasoning

TEACHING TIP: Review the properties of squares and equilateral triangles, as needed by the participants.

- Allow 7–10 minutes for the exercise.
- Get the participants' attention.
- Ask a volunteer to come to the front to write and explain the solution for each problem.
- Refer to *Transparency: Right Triangle Reasoning Answer Key* to resolve any questions.
- Point out to participants, if they do not mention it, that the diagonal of the square is the hypotenuse of the two isosceles right triangles it creates.
- Ask participants how they might generalize the solution to this problem to a rule that could be applied to any isosceles right triangle.
  - Draw a new isosceles right triangle on a blank transparency.
  - Label the length of the legs as a.
  - Ask participants to state the Pythagorean theorem using these measurements. ($c^2 = a^2 + a^2$)
  - Ask them what c is equal to. Insofar as possible, have participants direct the writing of the steps.

$$c^2 = a^2 + a^2$$
$$\sqrt{c^2} = \sqrt{a^2 + a^2}$$
$$c = \sqrt{2a^2}$$
$$c = (a)\sqrt{2}$$

- Remind participants that $\sqrt{2}$ (or the square root of any number not a squared number) is irrational.

- Review also the following concepts using questions to encourage participants to make the connections.
  - the relationship between the shortest side and the angle having the least measure (They are opposite each other.)
  - the relationship between the longest side and the angle having the greatest measure. (They are opposite each other.)

End of Pythagoras Who?

Mathematics, YES!

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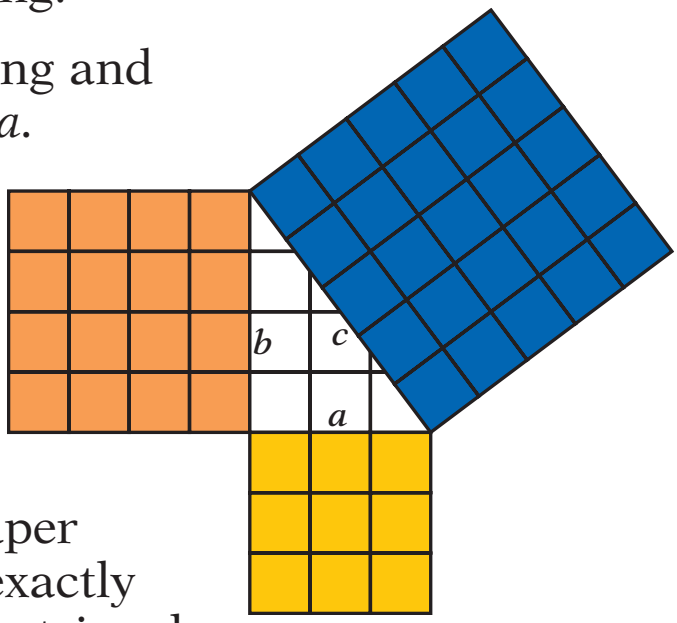
Right Triangles

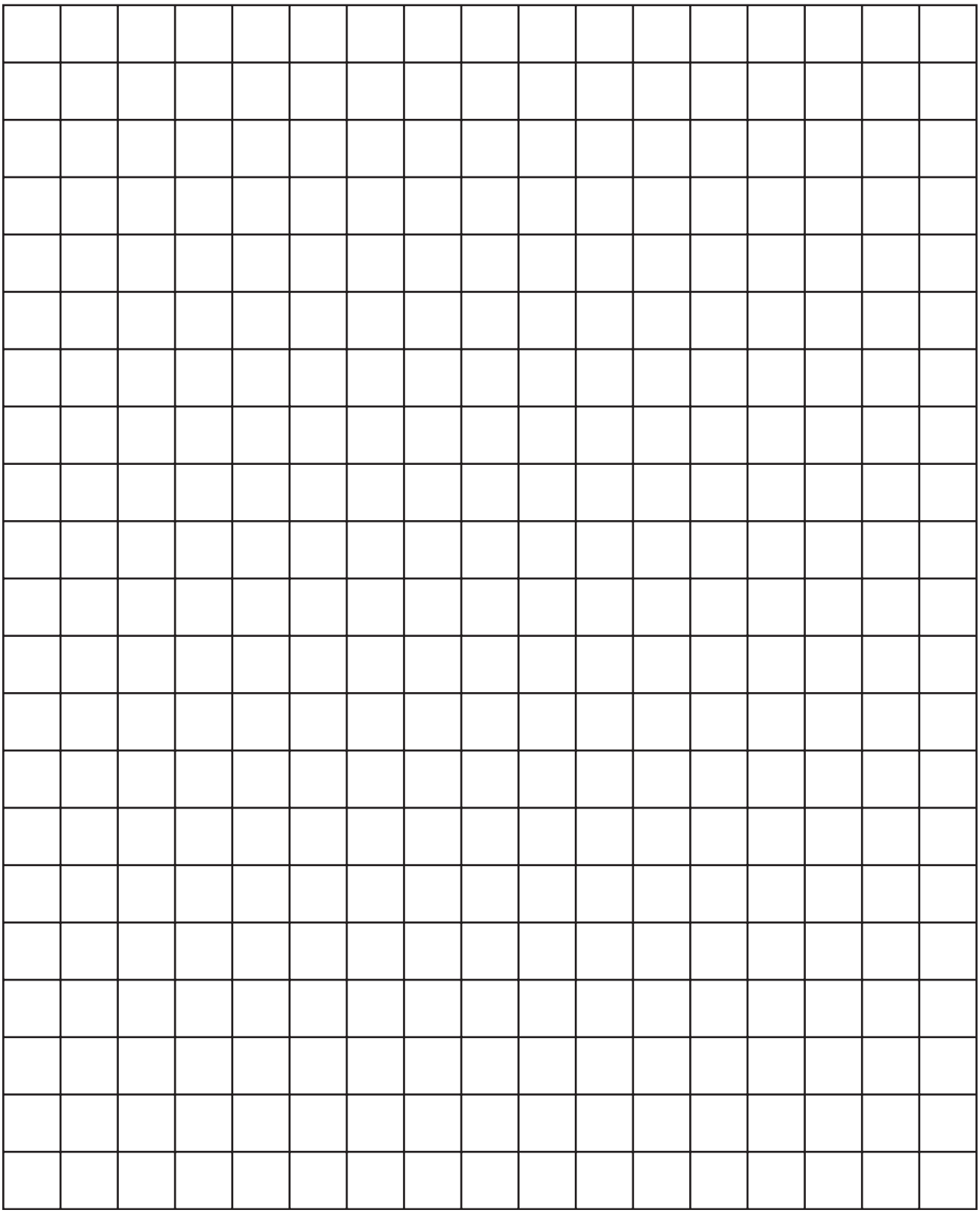
Directions

- Draw, on your grid paper, a right triangle as illustrated below.
- Make leg *a* 3 units long.
- Make leg *b* 4 units long and perpendicular to leg *a*.
- Connect the ends of the legs to create the hypotenuse, *c*.
- Cut out the triangle.
- Cut out three grid-paper squares that will fit exactly against the legs of the triangle.

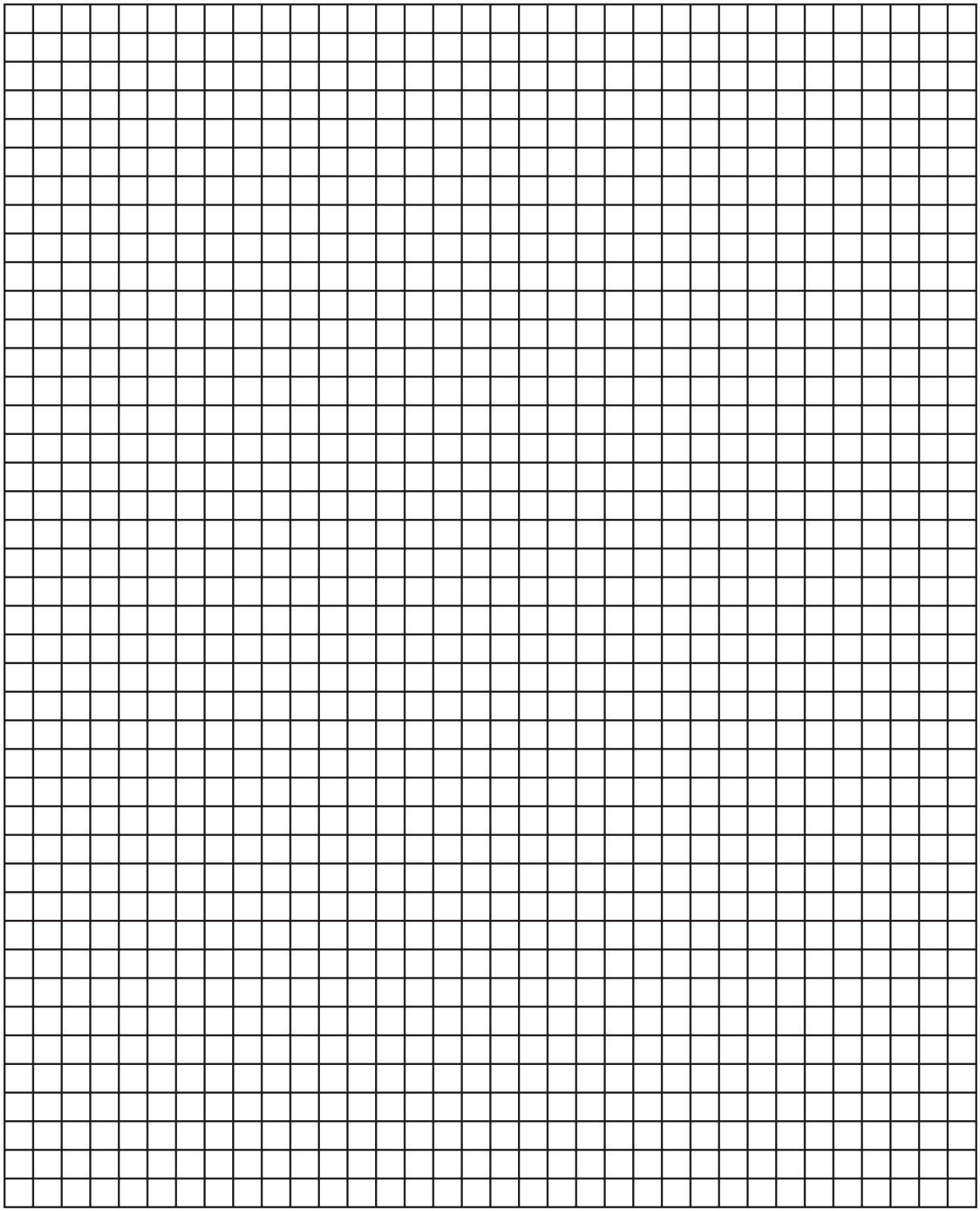


- Place the squares against the matching legs of the triangle.
- Find the area of each square and record the information on your *Right Triangle Relations Work Sheet*.
- Repeat the steps for triangles with perpendicular legs of:
- 8 units and 15 units
- 6 units and 8 units
- 5 units and 12 units

1-cm Grid



$\frac{1}{2}$-cm Grid



Right Triangle Relations

Length of sides (in units)	Areas of 3 squares	What is the relationship between the areas of the squares?	Is this true for all right triangles?
unit sides: **3 & 4** hypotenuse:			
unit sides: hypotenuse:			
unit sides: hypotenuse:			
unit sides: hypotenuse:			

Right Triangle Relations

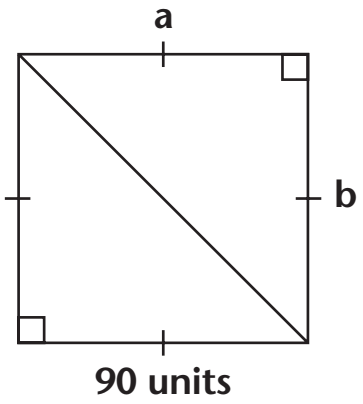
Answer Key

Length of sides (in units)	Areas of 3 squares	What is the relationship between the areas of the squares?	Is this true for all right triangles?
unit sides: **3 & 4** hypotenuse: **5**	**9** **16** **25**	The sum of the squares of the legs is equal to the square of the hypotenuse. $a^2 + b^2 = c^2$	**yes**
unit sides: **8 & 15** hypotenuse: **17**	**64** **225** **289**	The sum of the squares of the legs is equal to the square of the hypotenuse. $a^2 + b^2 = c^2$	**yes**
unit sides: **6 & 8** hypotenuse: **10**	**36** **64** **100**	The sum of the squares of the legs is equal to the square of the hypotenuse. $a^2 + b^2 = c^2$	**yes**
unit sides: **5 & 12** hypotenuse: **13**	**25** **144** **169**	The sum of the squares of the legs is equal to the square of the hypotenuse. $a^2 + b^2 = c^2$	**yes**

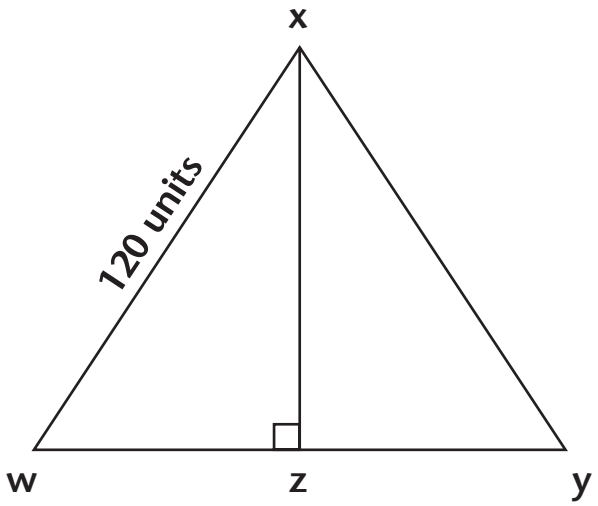
Right Triangle Reasoning

Use your knowledge of triangles and the Pythagorean theorem to calculate the elements requested below. Also, label the angles in each shape. Round your answers to the nearest tenth of a unit.

1. For the square, find the length of the diagonal.



2. For the equilateral triangle, find the length of the altitude.

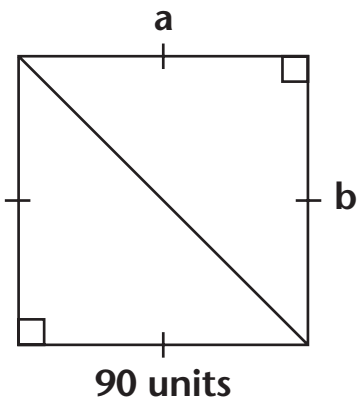


Right Triangle Reasoning

Answer Key

Use your knowledge of triangles and the Pythagorean theorem to calculate the elements requested below. Also, label the angles in each shape. Round your answers to the nearest tenth of a unit.

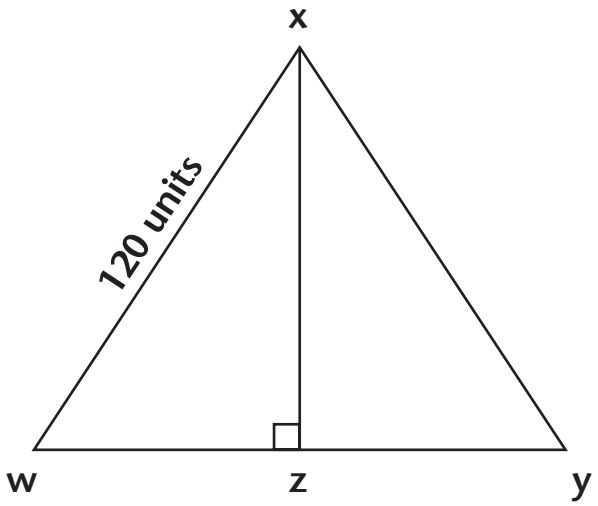
1. For the square, find the length of the diagonal.



Because sides a and b are the same:

$$c^2 = 2(a^2)$$
$$c^2 = 2(90^2)$$
$$c^2 = 2(8{,}100)$$
$$c^2 = 16{,}200$$
$$c = 127.3 \text{ units}$$

2. For the equilateral triangle, find the length of the altitude.



Because $\overline{wx} = \overline{wy}$

$$\overline{wz} = \overline{wy} \div 2$$
$$\overline{wz} = 120 \div 2$$
$$\overline{wz} = 60$$
$$\overline{xz}^2 = \overline{wx}^2 - \overline{wz}^2$$
$$\overline{xz}^2 = 120^2 - 60^2$$
$$\overline{xz}^2 = 14{,}400 - 3{,}600$$
$$\overline{xz}^2 = 10{,}800$$
$$xz = 103.9$$

Glossary

Geometry and Measurement of Plane Figures

acute angle — An angle with a measure less than 90 degrees (°).

angle — A geometric figure composed of two rays or line segments that share the same endpoint, called a vertex.

area — The number of square units in a region.

circle — The set of all points in a plane that are the same distance from a fixed point (the center of the circle).

circumference — The perimeter of (distance around) a circle. The circumference can be found using the formula $C = 2\varpi r$, where C is the circumference of the circle and r is the radius of the circle.

congruent figures — Two figures that have identical size and shape so that when one is placed over the other, they coincide exactly.

coordinate pair — An ordered pair of numbers that indicates the position of a point on a plane. The first number of a coordinate pair gives the point's location in relation to the x-axis. The second number in a coordinate pair gives the point's location in relation to the y-axis.

coordinate plane — A plane containing an x-axis and a y-axis. Every point on the plane can be described using a coordinate pair.

degree (°) — A unit of measure for angles. 1° is $\frac{1}{360}$ of a complete revolution around a point.

equilateral — The property of having equal, or congruent, sides.

equilateral triangle — A three-sided polygon with all sides and with all angles congruent.

hexagon — A six-sided polygon.

irregular polygon — A polygon in which not all the sides are congruent and not all the angles have the same measure.

Glossary (continued)

isosceles triangle	A triangle that has two congruent sides and two congruent angles.
line	The set of all contiguous (touching) points that form a straight path extending indefinitely in two directions opposite each other.
line segment	A part of a straight line that has two end points and a fixed length; a straight line segment marks the shortest distance between two points.
linear unit	A unit of measure for elements of a single dimension—length.
obtuse angle	An angle with a measure greater than 90° and less than 180°.
parallel lines	Lines that do not intersect and that are everywhere equidistant from each other.
parallelogram	A quadrilateral in which both pairs of opposite sides are parallel.
pentagon	A five-sided polygon.
perimeter	The distance around the outside of a plane shape or figure.
perpendicular	At right angles to. Two lines are perpendicular if their intersection creates right angles.
pi (ϖ)	The ratio of the circumference of any circle to its diameter (3.141592653 . . .). Pi is usually represented by the Greek letter, ϖ.
plane	A flat surface that extends forever in all directions.
plane figure	A figure that lies entirely in one plane.
point	A location in space.
polygon	A simple, closed plane shape composed of a minimum of three straight-line segments.

Glossary (continued)

quadrilateral	A four-sided polygon.
radius	A segment connecting the center of a circle to any point on the circle; the length of the radius.
ray	A subset of a line that includes one endpoint and that extends infinitely from that endpoint in one direction.
rectangle	A quadrilateral that includes four interior right angles.
regular polygon	A polygon in which all the sides are congruent and all the angles have the same measure.
rhombus	A parallelogram in which all sides are congruent.
right angle	An angle with a measure of 90°.
right triangle	A triangle with one right angle.
scalene triangle	A triangle in which no sides are congruent and no angles have the same measure.
similar figures	Figures that have congruent corresponding angles and in which corresponding sides are proportional.
square	A quadrilateral in which all sides and all angles are congruent.
square unit	A unit of measure used to describe the surface (area) of figures of two dimensions—length and width.
straight angle	An angle with a measure of 180°.
trapezoid	A quadrilateral in which only one pair of sides is parallel.
triangle	A three-sided polygon.
vertex (pl. vertices)	The intersection point shared by two sides of a polygon or the two sides (rays) of an angle. Also the intersection point shared by three or more edges of a polyhedron.